Efficiency of excitation of piezoceramic transducer at antiresonance frequency


Abstract. The efficiency of piezoceramic transducers excited at both the resonance and antiresonance frequencies was investigated. Losses in piezoceramics are phenomenologically considered to have three coupled mechanisms: dielectric, mechanical, and piezoelectric losses. Expressions for the resonance and antiresonance quality factors, which ultimately determine the transducer efficiency, have been received on the basis of complex material constants for both stiffened and unstiffened vibration modes. Comparisons of electric and mechanical fields, thermal and electrical losses of power supply, and their distribution in the transducer volume have then been made. For a given constant mechanical displacement of the transducer top, the required electric voltage applied to the transducer at the antiresonance frequency is proportional to the resonance quality factor, while the changes in the intrinsic electric and mechanical fields characteristics in the common case are not too essential. The requirements on the piezoceramic parameters, types of transducer vibration, and especially on the factor of piezoelectric losses in a range of physically valid values were established to provide maximal quality factors at the antiresonance frequency.

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I. INTRODUCTION.

Piezoceramic materials (PCM) of a PZT system [1] have unique electro-elastic properties. Piezoceramic transducers (PT) are used in electronics and hydro- and electroacoustics for a wide range of applications. High power mechanical output PT characteristics are traditionally achieved at the resonance frequency because of the relatively low requirement for electric voltage. However, the process of PT heating and, as a consequence, failure of output PT characteristics in strong electric and mechanical fields, restricts powerful PT applications. It requires optimization of regimes of PT excitation. As predicted in [2], the advantages of excitation of the resonant powerful PT at an antiresonance frequency, against the traditional resonant regime, with regard to PT heating and power supply losses, have found experimental and practical confirmations [3] - [6]. Conclusive specific and decisive roles of piezoelectric energy losses were determined.

The difference between the antiresonance $f_a$ and resonance $f_r$ PT frequencies [1], [8] depends on a degree of PCM polarization, namely on the value of the coefficient of electromechanical coupling (CEMC) $k_{ij}$ and harmonic number $n$ of a used type of vibration. The quality factor $Q$ characterizes resonant PT behavior. PZT-based PCM provide strong coupling of mechanical and electric fields (CEMC value can reach 0.9, close to theoretical limit [7]), so the mechanical and electrical components of energy losses, as well as the piezoelectric component describing losses at interconversion of the mechanical and electrical energies, should be involved in the phenomenological description of PT behavior. Traditionally, the resonance quality factor at the fundamental (lowest) harmonic of a PT planar mode (disk) [1], [8] was chosen for characterizing PCM quality.

Electro-elastic PT behavior caused by anisotropy of polarized piezoceramics generally is described by ten independent constants [8], [9]. Because it is an electro-mechanical oscillatory system, PT has two limit regimes of operation: short-circuit (s.c.) and open-circuit (o.c.) at closed and broken PT electrodes, respectively. The CEMC value is determined by a difference between real parts of the elastic PCM constants under a constant [1] electric field strength $E$ and induction $D$, corresponding in common case to the s.c. and o.c. regimes, in particular, $S^D_{33} = S^E_{33} (1 - k_{33}^2)$, where $k_{33}$ is the CEMC as $k_{33}^2 = d_{33}^2 / S^E_{33} e_{33}^T$. The CEMC as a real value defines a share of the total energy accumulated in PT and converted from mechanical into electrical forms and vice versa. In turn, the values of the quality factors corresponding to the s.c. ($Q_{s.c.}$) and o.c. ($Q_{o.c.}$) regimes in common case are defined by the complex elastic constants (e.g., $S^E_{33}$ and $S^D_{33}$), whose imaginary parts are defined by the mechanical, dielectric, and piezoelectric mechanisms of energy losses [9]. Taking into
account the piezoelectric component of losses for consideration was offered for the first time in [10]; the presence of a non-zero imaginary part of the piezocoefficient was found by direct measurement [11]; a high accuracy method of complex material constant determination was proposed in [12]; the physically valid limits on the piezoelectric loss factors was established in [9] according to the condition of “positivity” of local thermal losses in a piezoelectric media; the concept of complex material constants was used in [13], [14] to model dynamic PT response. Involving domain mechanism of energy loss [15] with damped (time delay) movement of walls of 90-degree domains, the uniform physical nature of coupled mechanical, dielectric, and piezoelectric energy loss components was shown.

In the common case, there are two basic types of PT vibrations: stiffened mode (SM) with direction of vibration along an exciting electric field (e.g., rod 15x2x2 mm PT (Fig. 3)) when the electric field induction is homogenous between electrodes [8], and unstiffened mode (UM) with direction of vibration perpendicular to the vector of an exciting electric field (e.g., bar 15x1x0.5 mm PT (Fig. 7)) when the electric field strength is homogenous between electrodes in the case of thin PT plate. Ultimately, research regarding the interrelation of the PT dissipative characteristics at the resonance and antiresonance frequencies – the resonance $Q_r$ and antiresonance $Q_a$ quality factors – is of practical and theoretical interest.

II. COMPLEX ELASTIC COMPLIANCES QUALITY FACTORS.

The electromechanical behavior of a real PT with energy losses generally is described [9] by: the complex constants of the PCM material matrix

$$S_{ij}^{E,D} = S_{ij}^{E,D} (1 - i Q_{ij}^{E,D}) , \quad \hat{d}_{kl} \equiv d_{kl} (1 - i \gamma_{kl}) , \quad \hat{\varepsilon}_{mn}^T \equiv \varepsilon_{mn}^T (1 - i \delta_{mn}) ;$$

(1)

the linear equations of piezoeffect

$$s_i = \hat{S}_{ij}^E T_j + \hat{d}_{ki} E_k , \quad D_n = \hat{\varepsilon}_{mn}^T F_m + \hat{\gamma}_{kl} T_l ;$$

and the equation of motion

$$\rho u_{tt} = \nabla T$$

at a given boundary and initial conditions, where $i \equiv \sqrt{-1}$, $Q_{ij}^{E,D}$ are the quality factors of the complex elastic compliances $\hat{S}_{ij}^{E,D}$, $\delta_{mn}$ and $\gamma_{kl}$ are the dielectric and piezoelectric loss angles, respectively, $\rho$ is the PCM density, $D$ and $E$, $s$ and $T$ are the electric field induction and strength, mechanical strain and stress, respectively, in a vector or tensor performance, $\nabla$ is the vector operator, $u$ is the local mechanical displacement. Similar performance is used for the constants of the material matrixes through $\hat{C}_{ij}^{E,D}, \hat{g}_{kl}, \hat{h}_{kl}, \hat{\beta}_{mn}^{T,S}$ [8], [9].
Particularly, the complex constants $\hat{\varepsilon}_{33}^D$ with their interrelation $\hat{\varepsilon}_{33}^D = \hat{\varepsilon}_{33}^E (1 - \hat{k}_{33}^2)$, where $\hat{k}_{33}$ is the complex CEMC, have “elastic constant quality factors” determined by the base relation:

$$
\frac{1}{Q_{33}^D} = \frac{1}{Q_{33}^E} - \frac{k_{33}^2}{1 - k_{33}^2} \left( 2\gamma_{33} - \delta_{33} - \frac{1}{Q_{33}^E} \right) = \frac{1}{Q_{33}^E (1 - k_{33}^2)} \left[ (1 - \sqrt{k_{33}^2 Q_{33}^E \delta_{33}})^2 + 2(1 - t_3) \sqrt{k_{33}^2 Q_{33}^E \delta_{33}} \right].
$$

(2)

From the condition of “positivity” of the local thermal energy losses, the limitation on the value of the piezoelectric loss angle was phenomenologically established [9] as follows

$$
|\gamma_{33}| \leq \gamma_3 \equiv \frac{2}{k_{33}^2 Q_{33}^E}, \quad \text{or} \quad t_3 \equiv \gamma_{33} / \gamma_3 \in [-1; 1].
$$

(3)

Under that condition, the “thermal local dissipation” is always positive, that means heating, so integral dissipation for the whole transducer is positive too, consequently. Meanwhile, it does not mean that each of the components causing the thermal losses, such as local losses of the electric or mechanical fields, must be positive separately in a piezoelectric body, especially with strong coupling of the electric and mechanical fields.

According to (2), $Q_{33}^D$ can be generally greater or less than $Q_{33}^E$. The maximum increase of the $Q_{33}^D / Q_{33}^E$ is provided by the non-zero imaginary part $\gamma_{33}$ of the piezocoefficient and corresponds to the conditions $k_{33}^2 Q_{33}^E \delta_{33} \to 1$ and $t_3 \to 1$. In a specific case of absence of the dielectric ($\delta_{33} = 0$) and piezoelectric ($\gamma_{33} = 0$) losses: $Q_{33}^D = Q_{33}^E (1 - k_{33}^2) \equiv Q_{33}^E (1 - 2\delta_r)$, where influence of the relative frequency resonant interval $\delta_r$ has opposite sign and stronger effect in comparison to the traditional PT equivalent circuit (EC), taking into account, as it is considered, only the mechanical losses [1] (see chapter VI).

Further, the following definition of the quality factor of a PT resonance (resonant peak) and accordingly a method for its calculation are used. Taking the expression describing the resonant PT behavior, after decomposition it by the small dissipative parameters ($1/Q_{\varepsilon}^{E,D}$, $\delta_{mn}$, $\gamma_{kl}$) and relative frequency displacement $\chi = f / f_0 - 1$ from the "ideal" frequency $f_0$ of the resonant peak (loss-free), this expression is performed [21] by first-order approximation as

$$
IA + \frac{B}{1 + i\xi},
$$

where A, B are coefficients, and the quality factor Q of the resonant peak is defined as a coefficient in the generalized frequency displacement $\xi = 2Q\chi$. The given approach methodically corresponds in full measure to the known frequency methods [1], [8] of quality factor determination, which are based on the measurement of characteristic frequencies of the real and imaginary parts of
resonant PT characteristics (e.g., admittance or impedance), and related to them methods [3], [4], [16], [17].

III. PT with STIFFENED VIBRATION MODE.

A. Resonance and Antiresonance PT Quality Factors

Let's further consider a “long rod” PT representing the SM vibration. The given type of longitudinal PT vibration along the vector of polarization is generally described by the complex electro-elastic PCM constants [1], [8], [9]: $\hat{S}_{33}^E$, $\hat{S}_{33}^D$, $\hat{\varepsilon}_{33}^T$, $\hat{\varepsilon}_{33}^T$, $\hat{\delta}_{33}$, $\hat{\gamma}_{33}$.

The PT complex impedance is the following:

$$Z = \frac{1}{k_0 \hat{C}_0 (1 - \hat{k}_{33}^2 \tan(Kh/2))} \left( 1 - \hat{k}_{33}^2 \frac{\tan(Kh/2)}{Kh/2} \right),$$  \hspace{1cm} (4)

where $\hat{C}_0$ is the quasistatic PT capacitance and $\hat{S}_{33}^E$, $\hat{S}_{33}^D$ are the quasistatic PT capacitance and expressions for the complex wavenumber and CEMC, respectively, $h$ is the PT length.

Using (4), after decomposition of the expressions for the PT admittance ($1/Z$) and impedance ($Z$) by the small parameters of the relative frequency displacement and dissipative coefficients in a vicinity accordingly of the resonance $f_{r,n} = B_n / 2h^{1/2} S_{33}^{D}$ and antiresonance $f_{a,n} = n / 2h^{1/2} S_{33}^{D}$ frequencies [21], where $B_n = 2q_n / \pi \sqrt{1 - k_{33}^2}$, $q_n$ is the $n$th-root of the frequency equation $	an q = q / k_{33}^2$ (e.g., $B_1 - 1 = 0.024$ at $k_{33} = 0.5$), we will receive in common case the expressions for the resonance $Q_{r,n}$ and antiresonance $Q_{a,n}$ quality factors of $n$-harmonic:

$$\frac{1}{Q_{r,n}} = \frac{1}{Q_{33}^E} - H_n \frac{k_{33}^2}{1 - k_{33}^2} \left( 2\gamma_{33} - \delta_{33,\text{in}} - \frac{1}{Q_{33}^E} \right) = \frac{1}{Q_{33}^E} \left[ 1 - H_n \frac{k_{33}^2}{1 - k_{33}^2} \left( 1 - k_{33}^2 Q_{33}^E \delta_{33,\text{in}} \right) \right]$$ \hspace{1cm} (5)

$$\frac{1}{Q_{a,n}} = \frac{1}{Q_{33}^D} - \frac{k_{33}^2}{1 - k_{33}^2} \left( 2\gamma_{33} - \delta_{33} - \frac{1}{Q_{33}^D} \right) = \frac{1}{Q_{33}^D} \left[ 1 - \frac{2(1 - k_{33}^2)}{q_n^2 - k_{33}^2 + k_{33}^4} \right] \left( 1 - k_{33}^2 Q_{33}^E \delta_{33,\text{in}} \right)$$ \hspace{1cm} (6)

where $H_n = 1 - \frac{2(1 - k_{33}^2)}{q_n^2 - k_{33}^2 + k_{33}^4}$. Note that the coefficient $H_n(k_{33})$ has values (at $k_{33} = 0...0.7$):

$H_1 = 0.18...0.10$, $H_2 = 0.91...0.95$, $H_5 = 0.97...0.98$, and further, $H_{n \to \infty} \to 1$.

For the fundamental harmonic ($n = 1$) we will further use the following approximation (no more 10%) for the coefficient $\frac{2(1 - k_{33}^2)}{q_1^2 - k_{33}^2 + k_{33}^4} \equiv \frac{8}{\pi^2}$, so that $H_1 \equiv 1 - 8 / \pi^2 = 0.18$. Thus, the resonance and antiresonance PT resistances are:
\[ R_\text{r}^{-1} = \frac{\omega C_0}{8 \pi} k_{33}^2 Q_r , \quad R_a = \frac{8 k_{33}^2 Q_a}{\pi^2 \omega_c C_0 (1-k_{33}^2)} . \] (7)

Here \( \delta_{33} = \delta_D + \delta_E (\delta_E = \delta_{E,\text{in}} + \delta_{E,\text{out}}) \), \( \delta_{33,\text{in}} = \delta_D + \delta_{E,\text{in}} \), \( |\gamma_{33}| \leq y_3 \equiv \sqrt{\delta_D/k_{33}^2 Q_{E}} \), \( t_3 = \gamma_{33} / y_3 \in [-1; 1] \), \( \delta_D \) is the dielectric (domain) component of the dielectric loss angle, \( \delta_E \) is the component caused by the conductivity of free charges (internal \( \delta_{E,\text{in}} \) and external \( \delta_{E,\text{out}} \); the last one can be PT surface conductivity or a resistor connected in parallel to PT).

In accordance with (5) and (6) for the fundamental harmonic \( (n=1) \) of our interest, the resonance quality factor \( Q_r = Q_{E}^E \) (taking into account \( H_l \ll 1 \) and \( Q_r \) has weak dependence on the other dissipative PCM parameters. At the same time, the range of variation of the antiresonance quality factor \( Q_a \) value can reach in common case some orders, that depends on a combination of the dissipative PCM parameters. The thermal (irreversible) PT energy losses \( W = 0.5 |V|^2 \text{Re}(1/Z) \) under an electric excitation by the voltage \( V \) are determined by the real part of the PT complex admittance. \( \text{Re}(1/Z) \) is the most sensitive to the piezoelectric loss component (\( t \)-parameter) at the frequencies above the resonance, including near-antiresonance frequency range (Fig. 1).

![Figure 1](image)

**Figure 1.** Calculated frequency dependence of the real part of the rod PT admittance for different values of \( t_{3} \)-parameter and \( Q_{E}^E =100 \), \( k_{33} = 0.7 \), and \( \delta_{33} = 0.015 \).

Calculated according to (4), the frequency dependence of the real part of the rod PT admittance at the fundamental harmonic is given in Fig. 1, which illustrates described PT dissipative properties at variation of the piezoelectric loss value in the presence of dielectric loss. The frequency dependence of the dissipation has a minimum (Fig. 2), which frequency matches the antiresonance at
$k_{33}^2 Q_{33}^E \delta_{33} \cong 1$ (we take here $\delta_{33}$ as $\delta_D$ component only). At $0.5 \leq t_3 \leq 1$ the total losses in the mentioned range of frequencies are less than the dielectric losses caused by the specific role of the piezoelectric component of losses [2], [18].

![Figure 2. Calculated frequency dependence of the real part of the rod PT admittance for different values of the dielectric loss parameter $\delta_{33}$ ($k_{33}^2 Q_{33}^E \delta_{33}$): curve 1 – 0 (0), 2 – 0.01 (0.49), 3 – 0.015 (0.74), 4 – 0.02 (0.98), 5 – 0.035 (1.72), at $Q_{33}^E = 100$, $k_{33} = 0.7$, and $t_3 = 1$ (for each curve). - - - - line of minima.](image)

**B. Basic Electric and Mechanical Field Characteristics**

The main characteristics of the rod PT vibration [1], [8] are described by:

\[
E_3(x) = \frac{V}{h} \left( 1 - \hat{k}_{33}^2 \right) N(x) F, \quad T_3(x) = \frac{V}{h} \frac{\hat{d}_{33}}{S_{33}^E} \frac{N(x) - 1}{-1 + \hat{k}_{33}^2 F}, \quad s_3(x) = \frac{V}{h} \frac{\hat{d}_{33} (1 - \hat{k}_{33}^2)}{-1 + \hat{k}_{33}^2 F} N(x),
\]

\[
u(x = h/2) = \frac{V}{2} \frac{\hat{d}_{33} (1 - \hat{k}_{33}^2)}{-1 + \hat{k}_{33}^2 F} F, \quad D_3(x) = \frac{V}{\omega Z \Delta} = \text{const}(x), \quad (8)
\]

where $F \equiv \frac{\tan(Kh/2)}{Kh/2}$, $N(x) \equiv \frac{\cos(x Kh/2)}{\cos(Kh/2)}$ are "resonant" factors, $x \in [0;1]$ is the relative coordinate from the center to the PT top, $U \equiv |u(x = h/2)|$ is the absolute value of the transducer top mechanical displacement.

Taking into account in common case that $k_{33}^2 Q_{33}^E \gg 1$ ("strong resonance") and considering just the fundamental harmonic ($n = 1$), we receive maximum values of the appropriate transducer characteristics. For the transducer top mechanical displacement ($x = 1$, absolute value):
\[ U(\omega_r) \equiv \frac{V d_{33}}{2} \frac{8}{\pi^2} Q_r, \quad U(\omega_a) = \frac{V d_{33}}{2} \frac{1-k_{33}^2}{k_{33}^2}, \quad U(\omega \to 0) = \frac{V d_{33}}{2}, \quad (9) \]

then \[ \frac{U(\omega_r)}{U(\omega_a)} = \frac{V(\omega)}{V(\omega_a)} \frac{8}{\pi^2} \frac{k_{33}^2}{(1-k_{33}^2)} Q_r. \]

For the mechanical stress (at the center \( x = 0 \) of PT, absolute value):
\[ \dot{T}(\omega_r) \equiv \frac{V d_{33}}{h S_{33}} \frac{4}{\pi} Q_r, \quad \dot{T}(\omega_a) = \frac{V d_{33}}{h S_{33}} \frac{\pi}{2} \frac{1}{k_{33}^2}, \quad \text{then} \quad \frac{T(\omega_r)}{T(\omega_a)} = \frac{V(\omega_r)}{V(\omega_a)} \frac{8}{\pi^2} k_{33}^2 Q_r. \quad (10) \]

For the mechanical strain (at the center \( x = 0 \) of PT, absolute value):
\[ \dot{s}(\omega_r) = \frac{V d_{33}}{h} (1-4k_{33}^2/\pi^2) \frac{4}{\pi} Q_r, \quad \dot{s}(\omega_a) = \frac{V d_{33}}{h} \frac{\pi}{2} \frac{1}{k_{33}^2}, \quad \text{then} \quad (11) \]
\[ \frac{\dot{s}(\omega_r)}{\dot{s}(\omega_a)} = \frac{V(\omega_r)}{V(\omega_a)} \frac{(1-4k_{33}^2/\pi^2)}{1-k_{33}^2} \frac{8}{\pi^2} k_{33}^2 Q_r. \]

For the maximum values of the electric field strength (at the center \( x = 0 \) of PT at the antiresonance \( \omega_a \) and at the top \( x = 1 \) of PT at the resonance \( \omega_r \) frequency, absolute values):
\[ \dot{E}(\omega_r) = \frac{V}{h} \frac{8}{\pi^2} k_{33}^2 Q_r, \quad \dot{E}(\omega_a) = \frac{V \pi}{h 2}, \quad \text{then} \quad \frac{\dot{E}(\omega_r)}{\dot{E}(\omega_a)} = \frac{V(\omega_r)}{V(\omega_a)} \frac{16}{\pi^3} k_{33}^2 Q_r. \quad (12) \]

Taking into account that the electric field induction \( D_3 = \text{const}(x) \) at any frequency (8) in the case of a rod PT, we have
\[ \frac{\dot{D}(\omega_r)}{\dot{D}(\omega_a)} = \frac{V(\omega_r)}{V(\omega_a)} \frac{\omega}{\omega_a} \frac{R_r}{R_a} \equiv \frac{V(\omega_r)}{V(\omega_a)} \left( \frac{8}{\pi^2} \right)^2 \frac{k_{33}^4 Q_r Q_a}{1-k_{33}^2}. \quad (13) \]

When the voltage applied to the PT does not depend on frequency, the maximum (along a PT length) value of the mechanical characteristics \( (T, s, u) \) has the resonance, the amplitude of which is proportional to the resonance quality factor \( Q_r \). At the same time, they do not have an antiresonance peak, as in the case of the electric field induction or total PT admittance. The electric field strength is inhomogeneous along the PT length and the characteristics of its distribution depend on the frequency of PT excitation. If, at the antiresonance frequency, the electric field strength has “sinusoidal” form of distribution along PT length, while at the resonance frequency the electric field reaches two local maxima at the center and at the top of a rod PT, proportional to the resonant factor \( k_{33}^2 Q_r \) (Fig. 3). That is the property inherent to the absolute value of the electric field strength and the following relationship must be anyway satisfied
\[ \Re \int_0^1 E_3(x) \, dx = V / h \quad (14) \]
For the energy loss efficiency under the same conditions, taking into account a common expression for the active energy losses of a generator of transducer excitation \( W = 0.5 |V|^2 \text{Re}(1/Z) \), we have:

\[
W(\omega_r) = \frac{V^2}{2R_r} \equiv \frac{1}{2} V^2 \omega_r C_0 \frac{8}{\pi^2} k_{33}^2 Q_r , \quad W(\omega_a) = \frac{V^2}{2R_a} = \frac{1}{2} V^2 \omega_a C_0 \frac{\pi^2}{8} \frac{1-k_{33}^2}{k_{33}^2 Q_a} ,
\]

then

\[
\frac{W(\omega_r)}{W(\omega_a)} \equiv \left(\frac{V(\omega_r)}{V(\omega_a)}\right)^2 \frac{\omega_r}{\omega_a} \left(\frac{8}{\pi^2}\right)^2 \frac{k_{33}^4 Q_r Q_a}{1-k_{33}^2} .
\]

If we consider the condition \( U(\omega_r) \approx U(\omega_a) \) (denoted by a vertical line) of our interest, then

\[
\frac{V(\omega_a)}{V(\omega_r)} = \frac{8}{\pi^2} \frac{k_{33}^2}{(1-k_{33}^2)} Q_r \approx (1...2) k_{33}^2 Q_r ,
\]

(the last estimation is given for the range of \( k_{33} = 0...0.7 \)). For the other characteristics we have

\[
\frac{\dot{T}(\omega_a)}{\dot{T}(\omega_r)} \equiv \frac{1}{1-k_{33}^2} \approx 1...2 , \quad \frac{\dot{s}(\omega_a)}{\dot{s}(\omega_r)} \equiv \frac{1}{1-4k_{33}^2/\pi^2} \approx 1...1.25 ,
\]

\[
\frac{\dot{E}(\omega_a)}{\dot{E}(\omega_r)} \equiv \frac{\pi}{2} \frac{1}{1-k_{33}^2} \approx 1.5...3 , \quad \frac{\dot{D}(\omega_a)}{\dot{D}(\omega_r)} \equiv \frac{\pi^2}{8} \frac{1}{k_{33}^2 Q_a} ,
\]

\[
\frac{W(\omega_r)}{W(\omega_a)} \equiv \frac{Q_r}{Q_a} \frac{\omega_r}{\omega_a} \frac{(1-k_{33}^2)}{Q_r} \approx \frac{Q_r}{Q_a} \frac{1}{(1...2.5)} .
\]

So, to provide equal amplitude of the PT top mechanical displacement at both the antiresonance and resonance frequencies, it is required that one applies a voltage to PT at \( \sim 2 k_{33}^2 Q_r \) greater...
(proportional to the resonance quality factor). At the same time, the maximum values (with respect to PT volume) of the mechanical strain and stress are no more than 1.3 and 2 times greater, respectively, at the antiresonance frequency, and these differences depend only on the CEMC value, but not dissipative characteristics (quality factor, etc.). Note that even if the voltage applied to a rod PT at the antiresonance frequency is too high \( \sim 2 k_{33}^2 Q_r \equiv 100 \) at \( k_{33} = 0.7 \) and \( Q_{33}^E = 100 \),

the maximum strength of the electric field (relative value) does not exceed 3 times and this ratio does not depend on quality factors. For this reason, non-linear effects at a strong level of PT excitation must be commensurable for the resonant and antiresonant regimes of PT excitation at equal PT top mechanical displacement, that allows to extend presented results to the practically important case of high power loading. At the same time, character of field distribution in the PT volume can differ significantly. The piezoelectric loss factor causes changes not only in total PT energy losses, but also influences the distribution of the thermal losses in the PT volume and can provide specific PT regions with an extremely low or high heating (Fig. 4).

\[ F_3 \] - parameter of piezoelectric losses. \( Q_{33}^E = 100, \ k_{33} = 0.7, \ \delta_{33} = 0.015. \) There are no piezoelectric losses at \( t_3 = 0. \)

The parameter (19) of relative energy losses of the electric source of PT excitation (total thermal losses in the whole PT volume), ultimately defining relative efficiency of PT excitation at the antiresonance frequency, are proportional to the ratio of the antiresonance quality factor to the resonance one with some coefficient depending only on the CEMC (relative resonant frequency interval). The expression and value of this factor depends on initial task in respect to the regime of
PT excitation. We have considered here the characteristics of the same PT at the resonance and antiresonance frequencies, whose relative difference depends only on the CEMC value and can exceed about 25…40% (for $k_{33}$ case). For this reason, the vibrational velocities have an additional difference by the same factor at equal amplitude of PT top mechanical displacement at the respective frequencies. For practical purposes, the comparison of two different PTs with equal resonance and antiresonance frequencies, respectively, easily can be made on the basis of the given relationships.

![Graphical representation](image)

**Figure 5.** Calculated dependence of total loss energy efficiency for the resonance and antiresonance frequencies on $t_3$ - parameter at equal top displacement of a rod PT. $Q_{33}^E = 100$ and $\delta_{33} (k_{33}^2 Q_{33}^E \delta_{33})$:

- curve 1 – 0 (0), 2 – 0.01 (0.49),
- 3 – 0.015 (0.74), 4 – 0.02 (0.98),
- 5 – 0.035 (1.72), all at $k_{33} = 0.7$,
- 6 – 0.044 (0.98) at $k_{33} = 0.47$,
- 7 – 0.083 (0.98) at $k_{33} = 0.34$.

Either way, for the total losses at the antiresonance frequency to be less than losses at the resonance frequency, the ratio of the respective quality factors must be at least $Q_a/Q_r > 1.5...2$, and for higher efficiency should essentially exceed this value (Fig. 5).

Functional dependences of the resonance $Q_r$ and antiresonance $Q_a$ quality factors on the mechanical, dielectric, and piezoelectric components of losses are described by (5) and (6). Taking into account that, for the SM vibration, $H_l << 1$ at the fundamental mode ($n = 1$), we have $Q_r = Q_{33}^E$ and $Q_a = Q_{33}^D$. From (5) and (6), follows that maximum values of the quality factors $Q_r$ and $Q_a$ correspond to the conditions

$$k_{33}^2 Q_{33}^E \delta_{33} D_1 \equiv 1 \quad \text{and} \quad t_3 \to 1 \quad .$$

(20)

The resonance quality factor $\max Q_r = Q_{33}^E/(1-H_1) \equiv Q_{33}^E \pi^2/8 \equiv 1.25 Q_{33}^E$. Theoretically $\max Q_a$ is unlimited, and the difference between $Q_a$ and $Q_r$ can reach some orders in an ideal case.

When considered traditionally, only the mechanical and dielectric mechanisms of energy losses causes a decrease (more than 2 times in respect to $Q_{33}^E$) in the antiresonance quality factor.
The possibility of an increase of the $Q_a$ quality factor value is a direct consequence of the presence of the non-zero imaginary part (at $t_3 > 0$) of the $\hat{d}_{33}$ piezoelectric coefficient. An increase of the $Q_a$ quality factor value is definitely impossible at $\delta_{33} \leq 0.1/k_{33}^2 Q_{33}^E$ and $\delta_{33} \Rightarrow 3.5/k_{33}^2 Q_{33}^E$ (Fig. 6).

From the experimental data for a PZT-based PCM, it follows that the received values of $t_3 = +0.5...0.95$ correspond to a maximum degree of polarization. In common case, the parameter $t_3$ is proportional to the CEMC $k_{33}$. According to the particular mechanism of energy losses [15], the damped movement of walls of 90-degree domains provides the $t_3$ - parameter, that is determined by only an effective angle $\alpha_0 \in (90...180)^\circ$ of domain orientation as $t_3 = (A_1 - A_3)/\sqrt{(1-A_2)(1-A_4)}$, where $A_m = \sin(m\alpha_0)/m\alpha_0$. We can estimate the character of change of the piezoelectric loss angle $\gamma_{33} = \hat{d}_{33}^*/\hat{d}_{33}'$, depending on PCM polarization ($\overline{P}$): $\hat{d}_{33}^* \sim \sin^3(\alpha_0)$ and $\hat{d}_{33}' \sim 1+\cos(\alpha_0)$. For $\overline{P} \rightarrow 0$ it follows, that $\gamma_{33} \rightarrow 0$ ( $Q_{33}^E$ and $\delta_{33}$ are supposed to be practically constant). However, the $\gamma_{33}$ value varies within limits of 20 % in a wide interval (1...0.1) of a saturated polarization (max $k_{33}$), i.e. for the mentioned interval of the CEMC values, it is possible to accept that $\gamma_{33} \approx \text{const (} \overline{P} \text{)}$, then

$$t_3 = k_{33} \gamma_{33} \sqrt{\delta_{33}/Q_{33}^E} \sim k_{33}. \quad (21)$$
IV. PT with UNSTIFFENED VIBRATION MODE.

A. Resonance and Antiresonance PT Quality Factors

Let's further consider a “long bar” PT representing the UM vibration. The given type of PT vibration in a plane perpendicular to the vector of polarization is generally described by the complex electroelastic PCM material constants [1], [8], [9]: $\hat{S}^{E,D}_{11}$ (quality factors $Q^{E,D}_{11}$), $\hat{\varepsilon}^{T}_{33}$ ($\delta_{33}$), $\hat{\lambda}_{31}$($\gamma_{31}$).

The complex dynamic admittance of the bar PT is described by:

$$Y = \imath\omega\hat{C}_0\left(1 - \hat{k}_{31}^2 + \hat{k}_{31}^2 \frac{\tan(\frac{Kb}{2})}{Kb/2}\right),$$

(22)

where $\hat{C}_0 (\hat{\varepsilon}^{T}_{33})$, $K^2 = \omega^2 \rho \hat{S}^{E}_{11}$, $\hat{k}_{31}^2 = \hat{\lambda}_{31}^2 / \hat{\varepsilon}^{T}_{33} \hat{S}^{E}_{11}$ are the quasistatic PT capacitance and the expressions for the complex wavenumber and CEMC, respectively, $b$ is the PT length.

After decomposition the expressions for the PT admittance ($Y$) and impedance ($1/Y$) by the small parameters of the relative frequency displacement and dissipative coefficients in a vicinity of accordingly the resonance $f_{r,n} = n/2b\sqrt{\rho S^{E}_{11}}$ and antiresonance $f_{a,n} = G_n/2b\sqrt{\rho S^{D}_{11}}$ frequencies [21], where $G_n = 2q_n\sqrt{1-k_{31}^2}/\pi$, $q_n$ is the $n^{th}$-root of the frequency equation $\tan q = -q (1-k_{31}^2)/k_{31}^2$ (e.g., 1– $G_1 = 0.031$ at $k_{31} = 0.5$), we will get the expressions for the resonance $Q_{r,n}$ and antiresonance $Q_{a,n}$ quality factors ($n = 1, 3, 5...$ - harmonic number):

$$Q_{r,n} = Q^{E}_{11},$$

(23)

$$\frac{1}{Q_{a,n}} = \frac{1}{Q^{E}_{11}} - F_n \frac{k_{31}^4}{1-k_{31}^2} \left(2\gamma_{31} - \delta_{33} - \frac{1}{Q^{E}_{11}}\right) = \frac{1}{Q^{E}_{11}} \left(1-F_n + \frac{F_n}{1-k_{31}^2} \left[1 - \sqrt{k_{31}^2Q^{E}_{11}\delta_{D}}\right]^2 + 2(1-t_1)\sqrt{k_{31}^2Q^{E}_{11}\delta_{D}} + k_{31}^2Q^{E}_{11}\delta_{E}\right),$$

(24)

where $F_n = \frac{2(1-k_{31}^2)}{q_n^2-(2q_n^2-1)k_{31}^2+q_n^2k_{31}^4}$. Note that the coefficient $F_n \equiv F_n (k_{31})$ has values:

$F_1 = 8/\pi^2 (=0.81)...0.75$ at $k_{31} = 0...0.5$, and further, $F_{n\to\infty} \sim 1/n^2$. Here $|\gamma_{31}| \leq y_1 = \sqrt{\delta_{D}/k_{31}^2Q^{E}_{11}}$, $t_1 = \gamma_{31}/y_1 \in [-1; 1]$. For the fundamental harmonic ($n = 1$) the resonance $R_r$ and antiresonance $R_a$ PT resistances are described by (7) with replacement of $k_{33} \to k_{31}$ parameters.

Thus, the resonance quality factor $Q_r$ value at the fundamental mode is determined exactly by the quality factor $Q^{E}_{11}$ of the complex elastic compliance $\hat{S}^{E}_{11}$, $Q_r = Q^{E}_{11}$.

Taking into account, that for the UM vibration $F_1 = 1$ ($n = 1$), we have $Q_d \equiv Q^{D}_{11}$. From (24), it follows, that maximum value of the antiresonance quality factor $Q_a$ at the fundamental mode corresponds to the conditions
\( k_{31}^2 Q_{11} E S_{33(D)} = 1 \) and \( t_1 = 1 \), thus \( \max Q_a = Q_{11}^E / (1 - F_1) \geq 5.3 Q_{11}^E \). Theoretically in an ideal case the difference between \( Q_a \) and \( Q_r \) can be extremely appreciable.

**B. Comparative Analysis of the Loss Distribution in a PT Volume for the Stiffened and Unstiffened Modes of Vibration.**

The resonance quality factor \( Q_r (n = 1) \) of the UM vibration is only determined by the mechanical loss component (\( Q_{11}^E \) for a bar PT), but, for the SM vibration, it depends on both \( Q_{33}^E \) (rod PT) and, to some degree, the dielectric and piezoelectric loss components. The influence of the last components takes place because, in the case of SM vibration, the local electric field \( E \) is inhomogeneous between the PT electrodes and has resonant character (similarly to current, deformation, etc.) under the condition (14). The local value of the field strength \( E \) at the resonance increases by orders that, as a consequence, increases a share of the dielectric (and piezoelectric) loss component in the total resonance energy loss. In particular, for this reason, the contribution of conductivity of free volume charges into the value of the resonance quality factor increases (mainly in the PT volume regions with large resonant values of the field \( E \)), but there is no contribution of surface or external conductivity where field \( E \) has normal, non-resonant value. At the same time, in the case of UM vibration, the electric field strength has relatively low value due to its non-resonant behavior.
Relative properties of the mechanical field (strain, stress) in the case of bar PT (UM) are described by the relationships similar to (17) for the rod PT case (SM). However, the distributions of the electric field strength and induction differ essentially. The electric field strength \( E = \text{const}(f) \) does not depend on frequency and has homogeneous distribution in bar PT volume. The electric field induction has inhomogeneous distribution along bar PT length (frequency-specifying size), has also resonant (max \( \sim k_3^3 Q_r \)) and antiresonant (min \( \sim 1/k_3^3 Q_a \)) peaks, and provides specific volume distribution of both local losses of electric source of excitation and local thermal losses (Fig. 7, 8).

The local losses of the electric field are described by \( p(x) = 0.5E_3 \Re(j^*(x)) = 0.5\omega E_3 \Im(D_3^*(x)) \), where \( j(x) \) is the current density (* - complex conjunction), then the total in PT volume \( \Omega \) energy loss of power supply is \( P = \int_\Omega p(x)d\Omega = 0.5|V|^2 \Re(1/Z) \) and equals the total PT thermal loss. When the value of \( t_1 \)-parameter of the piezoelectric loss lies in the interval \( t_1 = 0.8...1 \), the energy losses of the electric field in some frequency range, including the antiresonance frequency, are negative at the center region of the bar PT (Fig. 7). It means that the current at the PT center is opposite to the applied voltage, i.e. the local “negative dynamic conductivity” [2], [18] of the central region of the bar PT takes place. At the same time, the local thermal dissipation is always positive at any point of the PT volume (Fig. 8).

![Figure 8. Calculated distribution of local thermal losses along length of a bar PT at the resonance (r) and antiresonance (a) frequencies under the condition of a constant PT top displacement (U) for: \( Q_{11}^E =100 \), \( k_3 = 0.7 \), \( \delta_{zz} = 0.02 \) and \( t_{11} \)-parameter in the allowed range [-1, 1] (shown in parentheses).](image)

The reason of such difference in distribution of the local thermal and electric field losses depends on redistribution (energy flux) of coupled electro-mechanical energy because of the concept of the Umov-Pointing vector [19]. Particularly in the case of bar PT
where \( w(x) \) is the local thermal loss. As PT is supposed to be electrically excited only, according to the boundary conditions, the total in PT volume irreversible flux of the “mechanical” energy

\[
\int_{x=0}^{x=\Omega} \text{Im}(T_i s_i^*) \, d\Omega = 0 .
\]

Note that the function of distribution of the thermal losses at the antiresonance frequency has two maxima at the center and top of the bar PT at positive values of \( t_1 \)-parameter. The last feature is very important for a comparative experiment on the determination of the efficiency of two regimes of PT excitation and must be taken into account. If a temperature (heat) detector is located only at the PT center [3], the error in total energy heating measurement can be double, and incorrect conclusion on the energy efficiency could be made.

The specific features of the considered bar PT behavior and the basic quality factors relationships (23) and (24) are similar to a disk PT of radial (planar) vibration (UM). Taking into account that CEMC \( k_p \gg k_{31} \) and that the \( t_{(p)} \)-parameter of piezoelectric losses is proportional to CEMC, described effects have to take place to a greater degree in the case of disk PT. Calculated according to (23) and (24), the dependence \( Q_a / Q_r \) \((n = 1)\) versus \( k_p Q_{E1}^E \delta_{33} \) for various values of the generalized F and \( t \) parameters is shown in Fig. 6, and is applied both to the bar and disk PT.

Commonly, the quality factors, such as \( Q_r \) of the resonance PT peak, \( Q_{E1}^E \) of appropriate elastic constant, and \( Q_{s.c.} \) corresponding to the short-circuit regime, are approximately equal to each other at least for the fundamental harmonic (the same property takes place for the set of the quality factors \( Q_a \), \( Q_{E1}^D \) and \( Q_{o.c.} \) at antiresonance). The difference between them inside each set depends on the type of vibration, harmonic number, etc. They are exactly equal just for a PT with “concentrated parameters”, such as thin ring resonator, when all local resonant mechanical and electrical characteristics (deformation, strength, etc.) are homogeneous in the PT volume.

V. EXPERIMENTAL RESULTS on QUALITY FACTOR RELATIONSHIPS.

According to (23) and (24), the character of the difference between \( Q_r \) and \( Q_a \) is stipulated by the piezoelectric dissipative effect caused by the non-zero imaginary part of the piezocoefficient, the difference increases with CEMC growth. The experimental dependence of the disk PT quality factors on a degree of polarization \((k_p (\delta_r))\) is shown in Fig. 9. The antiresonance \( Q_a (\delta_r) \) relationship has the strongest dependence, which increases with growth of polarization. For the maximum achieved \( k_p = 0.55 \) \((\delta_r = 0.15)\), the ratio \( Q_a / Q_r \) reaches more than 2, at \( k_p \to 0 \) \( Q_a \to Q_r \to Q_0 \) to
the quality factor of a non-polarized PCM. Meanwhile, the $Q_r(\delta_r)$ curve reflects the known fact [1] that $Q_r$ decreases insignificantly as PCM polarization increases.

Figure 9. Influence of the relative resonant interval $\delta_r$ of a disk PT with planar mode of vibration on the resonance $Q_r$, antiresonance $Q_a$ quality factors, and their ratio $Q_a/Q_r$:

- lines 1, 2 - interpolation of the experimental values of $Q_r$ (1) and $Q_a$ (2),
- lines 3, 4 – calculated dependences $Q_a/Q_r(\delta_r)$ according to (24) at $Q_0 = 600$, $\delta_{33} = 0.007$ and accordingly: 3 - max $t_p = 1$ (at $k_p = 0.63$), 4 - $t_p = 0$ ($\gamma_{31} = 0$).

PCM PZT-35Y.

Calculated dependence $Q_a/Q_r$ for a planar mode of vibration in view of the conditions $\gamma_{31} = \text{const} (\delta_p)$ and $t_p \sim k_p$ (21) in the researched range of polarization has shown a satisfactory agreement of theoretical and experimental data, and $t_p \equiv 0.90$ was observed at maximum achieved CEMC value. The absence of the imaginary part of the piezocoefficient ($\gamma_{31}, t_p = 0$) should have resulted in the converse effect – decreasing of $Q_a/Q_r$ more than 2 times.

Rhombohedral, morphotropic and tetragonal [1] PCM compositions were involved into the experimental research. For the disk PT with radial mode of vibration the maximum $Q_a/Q_r$ value corresponds to the fundamental harmonic ($n = 1$), and the ratio $Q_a/Q_r (n = 1)$ reaches 1.8...2.4, while the ratio $Q_a/Q_r (n = 1)$ can not exceed 3.0 for max $F_1 = 0.8$ and $t_p = 0.95$ according to (24).

It is necessary to take into account, that the dielectric loss angle consists of two components $\delta_{33} = \delta_D + \delta_E$, defined accordingly by the domain mechanism of energy losses $\delta_D$ (which determines $t_p (t_1, t_3)$ value), and by the conductivity of free charges $\delta_E$. The last component adds additional term to the expressions (5), (6) and (24) resulting in decreasing of $Q_a/Q_r$. 

\[ \delta_{33} = \delta_D + \delta_E \]
The experimental data presented in Fig. 10 show the dependence of the resonance $Q_r$ and antiresonance $Q_a$ planar quality factors at the fundamental mode of the disk PT on the factor of dielectric losses $\tan \delta$ (1 kHz). The PT samples were made of a single block 60x60x8 mm of PCM PZT-35Y (Russia), as a representative of filter and acoustic PCM, sintered at a non-optimal regime with a large gradient of properties. The samples were polarized in air under pressure at temperature transition through Curie point. At enough equal degree of polarization of the samples (planar $\delta_r = 10\ldots14\%$), the lowering of density, steep increase of the static conductivity and increased porosity took place with increase of $\tan \delta_{(33)}$. Under the effect of the indicated factors, the resonance quality factor of planar vibration is reduced a little bit, the planar antiresonance quality factor decreases steeply, that corresponds to the theoretical analysis. There is one important practical conclusion, that the change of the antiresonance quality factor $Q_a$ value is a sensitive indicator of the internal active conductivity connected with the internal micro-defectiveness of PCM structure, such as porosity, not reacted metal components or inclusions, etc. In most cases, the additional firing (700 - 900 $^\circ$C) of samples with increased static conductivity is enough for essential increasing of the antiresonance quality factor, when "healing" of micro-cracks and partial “burning out” of conducting inclusions at increased porosity take place.
VI. QUALITY FACTOR PROPERTIES OF PT EQUIVALENT CIRCUIT.

The traditional PT equivalent circuit [1], [20], [21], representing an electrical analog of the PT electromechanical system with the typical elements \( R, L, C_d, C_s \) \( (C_0 = C_d + C_s) \), does not allow one to predict a real difference in the quality factor values at the resonance and antiresonance frequencies. When a single resistive element is used (reflecting the mechanical losses in PT, as it is traditionally considered), the antiresonance quality factor value is greater than the resonance one (on the value of relative resonant interval only). Taking into account the dielectric losses as an additional resistive element \( r_d \), connected in parallel to the EC \( \tan \delta = 1/r_d \omega_r C_0 \), the value of the antiresonance quality factor is reduced as \( Q_a = Q_r (1 + \delta_r)/(1 + 2 \delta_r Q_r \tan \delta) \), where \( \delta_r = (\omega_a - \omega_r)/\omega_r \).

For better performance, in a "new" EC [20] it is offered to connect the additional resistor \( r_s \) in series to the "traditional" EC. Presence of the two resistive elements \( R \) and \( r_s \) (actually independent for series (resonant) and parallel (antiresonant) EC partial circuits) allows to change the EC \( Q_r \) and \( Q_a \) independently and over a wide range of values, i.e. to closer match the EC behavior with the real PT properties according to the experimental data. The theoretical explanation of such a practical circuit was presented in [21], where the dominant piezoelectric mechanism of energy losses was involved.

Using the method of decomposition by the small parameters of the expressions describing the admittance of the bar PT and its "traditional" EC, from the condition of their equality, the dependence of the EC equivalent resistance \( R \) on the relative resonant frequency displacement \( \chi = f/f_r - 1 \) inside the resonance-antiresonance interval was obtained:

\[
R(f) = R(f_r) \left[ 1 - 2 Q_{11}^E \chi \left( 2 \gamma_{31} - \frac{3}{2 Q_{11}^E} - \frac{\chi}{\delta_{33}} \right) \right],
\]

\[
R(f_a) = R(f_r) \left[ 1 - 2 \delta_r Q_{11}^E \left( 2 \gamma_{31} - \frac{3}{2 Q_{11}^E} - \delta_{33} \right) \right],
\]

that allows to determine the EC dissipative elements with piezoelectric factor included.

Then, for the “new” EC with fixed resistive elements \( R \) and \( r_s \), their values are defined as

\[
r_s = \left( 2 \gamma_{31} - \frac{3}{2 Q_{11}^E} - \delta_{33} \right)/\omega_r C_0 \quad \text{and} \quad R = 1/\omega_r C_0 k_{33}^2 Q_{11}^E - r_s .
\]

VII. CONCLUSION.

Comparative analysis of the energy efficiency of PT excitation at both the antiresonance and resonance frequencies was made taking into account the basic mechanical, dielectric, and piezoelectric mechanisms of energy losses. It was established that the character of the interrelation of the resonance and antiresonance PT quality factors is determined mainly by the imaginary part of
the piezocoefficient whose influence is the most essential at the fundamental antiresonance frequency for both SM and UM vibrations. Greater value of the antiresonance quality factor in respect to the resonance quality factor is provided under the following conditions: for the PCM and PT-type with the greatest CEMC value $k$ (such as $k_{33}$ and $k_p$ for a rod and disk PT), the combination of the electro-elastic parameters has to be near $k^2 Q_{(r)} \delta_{(D)} \approx 1$ with maximal value up to +1 of the relative $t$-parameter of the piezoelectric losses, minimal conductivity of free volume charges.

To provide equal PT top mechanical displacement amplitude at the antiresonance and resonance frequencies, it is required that one applies a voltage to the PT $\sim 2 k_{33}^2 Q_r$ times greater (proportional only to the resonance quality factor) at the antiresonance frequency, than at the resonance frequency. At the same time, maximum values of the mechanical strain and stress in the PT volume are no more than 1.3 and 2 times greater, respectively, at the antiresonance frequency, and these differences depend only on CEMC, but not dissipative characteristics (quality factor, etc.). Even if the voltage applied to the PT with SM vibration (rod PT) at the antiresonance frequency is too high and typically exceed some orders, the maximum electric field strength (relative value for two regimes considered) does not exceed 3 times and this ratio does not depend on quality factor. The piezoelectric loss factor causes changes not only in the total PT energy losses, but also influences on the distribution of the thermal losses in the PT volume and can provide specific PT regions with an extremely low or high heating. For the physically valid positive values of $t$-parameter of the piezoelectric losses, the local electrical energy losses of the exciting electric field at the antiresonance frequency can be negative at the center region of a thin PT of UM vibration, when local current at the PT center is opposite to the applied voltage (“negative local dynamic conductivity”). At the same time, local thermal dissipation is always positive at any point of the PT volume.

Presented results will be useful for the optimization of regimes of PT excitation to provide higher efficiency. Described approach can be successfully applied to the analysis of the PCM quality factors for other PT applications where energy dissipation is the most critical, and can be used in the methods of prediction of the PT properties in different conditions [22]. Taking into account a wide range of values (up to 3 times) of quality factors for at least main types of PT vibrations, specification of the concept of PCM "mechanical quality factor" is obviously necessary.

Since the influence of the piezoelectric component of energy losses increases with growth of the CEMC value, that can be essential factor in description of the properties of a new class of strong piezoelectric materials ($k_{33} \approx 0.95$), such as PMN-PT [7].
REFERENCES


Glossary

\( Q, Q_q^{E,D} \) – generalized quality factor and quality factors of the complex elastic compliances \( \hat{S}_{ij}^{E,D} \)

\( Q_{s.c.}, Q_{o.c.} \) – PT quality factor corresponding to the s.c. and o.c. regimes of PT excitation

\( Q_{r,n}, Q_{a,n} \) – resonance and antiresonance PT quality factors of \( n \)-harmonic

\( \hat{S}_{ij}^{E,D}, S_{ij}^{E,D}, \hat{d}_{ij}, \hat{d}_{ij}^T, \hat{\xi}_{mn}, \hat{\xi}_{mn}^T \) – piezomaterial constants (complex and real values)

\( \hat{k}_{ij} \) – coefficient of electro-mechanical coupling (complex and real values)

\( \chi, \xi \) – relative and generalized frequency displacement (\( \chi = f / f_r - 1, \xi = 2Q\chi \))

\( q_n, K \) – root of a frequency equation and complex acoustic wavenumber

\( f_r(n), f_a(n) (\omega_{r,n}, \omega_{a,n}) \) – resonance (r) and antiresonance (a) frequencies of \( n \)-harmonic (real values)

\( \delta_r \) – relative resonance frequency interval (\( \delta_r = f_a / f_r - 1 \))

\( Y, Z \) – PT admittance and impedance

\( R_r, R_a \) – resonance and antiresonance PT resistances
\( \hat{C}_0(C_0) \) – PT capacitance (complex and real values)

\( \delta_{mn} (\delta, \delta_D, \delta_E, \tan\delta) \) – dielectric loss angle factor

\( \gamma_{kl} \) – piezoelectric loss angle

\( y_1, y_3 \) – limit values of piezoelectric loss angle

\( t_1, t_3 \) – normalized piezoelectric loss factors (e.g., \( t_1 \equiv \gamma_{31} / y_1 \))

\( h, b \) – frequency-specifying PT dimension

\( \Omega, \Delta \) – PT volume and electrode area

\( H_n, F_n \) – piezoelectric parameters of the PT quality factors relationships

\( B_n, G_n \) – frequency “correction” coefficients

\( x \) – relative local coordinate \([-1;1]\]

\( A_m, \alpha_0 \) – parameter and effective angle of domain orientation

\( R, L, C_d, C_S (C_0) \) – traditional parameters of the PT equivalent circuit

\( V \) – electric voltage applied to PT

\( D_{(3)} \) and \( E_{(3)}, T_{(3)} \) and \( s_{(3)} \) – electric field induction and strength, mechanical stress and relative deformation

\( u(x), U \) – local and PT top displacement

\( w(x), W \) – local and total PT thermal losses

\( p(x), P \) – local and total PT losses of electric field (power supply)

\( M \) – parameter of specific dielectric loss

Field or dissipative parameter with point on the top (e.g., \( \hat{T} \)) - parameters’ maximum absolute value in the PT volume

**Abbreviations**

PCM, PT – piezoceramic material and piezoceramic transducer

SM, UM – stiffened and unstiffened mode of vibration

CEMC – coefficient of electro-mechanical coupling

EC – PT equivalent circuit

s.c., o.s. – sort-circuit and open-circuit regime of PT excitation

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